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AIAA 81-4034

Transonic Solution for Nieuwland Profiles Using Spline Interpolation

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I. Introduction

USING the indirect hodograph method, Nieuwland[1] obtained exact shock-free solutions for a number of quasielliptical airfoil sections. But the hodograph method has the disadvantage that the body shape cannot be prescribed a priori, so the boundary condition has to be satisfied at unknown boundaries.

The direct problem where the profile shape and the freestream Mach number M_∞ (<1) are prescribed is of much practical interest. We consider here the case of a thin symmetric profile at zero incidence. The integral equation formulation of this problem[2] leads to a two-dimensional nonlinear singular integral equation for the unknown u component of the velocity parallel to the freestream direction. This is known as the integral equation of Oswatitsch. Niyogi[3,4] obtained an approximate solution of the above integral equation for shock-free profile flow where the transonic solution is expressed in terms of the corresponding linearized Prandtl solution. Comparison with other results indicated good agreement.

However, it turns out that the profile geometry of most body shapes of practical interest is given numerically. In computing the linearized Prandtl solution, the profile slope is needed, which then has to be evaluated numerically. Furthermore, a singular integral remains to be computed numerically. In general, this leads to loss of accuracy. To overcome this, in the present work, the profile shape has been represented by cubic splines, which has the advantage that the profile slopes are derived with adequate accuracy. Moreover, the integration needed for evaluating the linearized Prandtl solution can be performed analytically.

In the present work, results have been computed for a number of symmetrical quasielliptical Nieuwland profiles at zero incidence, for which exact solutions are known.[1] An edge correction has been used in the linearized solution. Excellent agreement has been found in all cases.

II. Formulation of the Problem

We consider steady inviscid transonic flow past a thin symmetric profile at zero incidence, with subsonic freestream Mach number $M_\infty<1$. According to integral equation formulation, the flow problem in the shock-free case is governed by the following two-dimensional nonlinear singular integral equation[2,3]:

$$U(X,Y)=U_p(X,Y)+\frac{U^2(X,Y)}{4}-\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\times\frac{U^2(\xi,\eta)}{2}\frac{(\xi-x)^2-(\eta-y)^2}{[(\xi-x)^2+(\eta-y)^2]^2}\mathrm{d}\xi\mathrm{d}\eta \tag{1}$$

Here, reduced rectangular Cartesian coordinates X,Y and reduced velocity components U,V are related to their true values, indicated by x,y,u,v as follows:

$$X=x,\ Y=y\sqrt{1-M_\infty^2},\ U=\frac{u-u_\infty}{c^*-u_\infty},\ V=\frac{v}{(c^*-u_\infty)\sqrt{1-M_\infty^2}} \tag{2}$$

c^* being the critical sound speed. $U_p(X,Y)$ is the linearized Prandtl solution defined by

$$U_p(X,Y)=\frac{1}{\pi}\int_0^1\frac{V_0(\xi)(X-\xi)}{(X-\xi)^2+Y^2}\mathrm{d}\xi \tag{3}$$

Niyogi[4,5] obtained an approximate solution of the singular integral equation (1), for shock-free flow as

$$U(X,Y)=(\sqrt{3}+1)[1-\{1-(\sqrt{3}-1)U_p(X,Y)\}^{1/2}] \tag{4}$$

Using the boundary condition at the profile

$$V(X)=V(X,0)=T\frac{\mathrm{d}f(X)}{\mathrm{d}X} \tag{5}$$

where T is the reduced thickness ratio, related to the thickness ratio τ as

$$T=\frac{\tau}{(1/M_\infty^*-1)\sqrt{1-M_\infty^2}} \tag{6}$$

Equation (3) yields

$$U_p(X,Y)=\frac{A}{\pi}\int_0^1\frac{\{\mathrm{d}h(X)/\mathrm{d}X\}(X-\xi)}{(X-\xi)^2+Y^2}\mathrm{d}\xi \tag{7}$$

where

$$A=\frac{1}{(1/M_\infty^*-1)\sqrt{1-M_\infty^2}};\quad h(x)=\tau f(x)$$

$h(x)$ is the profile shape.

The problem arises when $h(x)$ is not given analytically, and instead is prescribed by a set of numerical data. Our natural choice was then spline interpolation, which is capable of delivering results of adequate accuracy. Given a set of N mesh points $\{(x_i,y_i),\ i=1,\ldots,N\}$ describing the continuous profile shape, a cubic polynomial is chosen for the ith interval as

$$y=\alpha_i(x-x_i)^3+\beta_i(x-x_i)^2+\gamma_i(x-x_i)+\delta_i \tag{8}$$

Constants α_i, β_i, γ_i and δ_i are evaluated,[6] using the property that the cubics and their first and second derivatives are continuous (i.e., the condition to be required that both the slope, $\mathrm{d}y/\mathrm{d}x$, and the curvature, $\mathrm{d}^2y/\mathrm{d}x^2$, are the same for the pair of cubics that join at each point) at the pivotal points. The linearized solution $U_p(X,Y)$ is then found by simple integration as

$$U_p(X,Y)=\sum_{i=1}^{N}-\frac{1}{\pi A}\left[\frac{3\alpha_i}{2}\{(X_{i+1}-X)^2-(X_i-X)^2\}\right.$$

Received Dec. 27, 1979.
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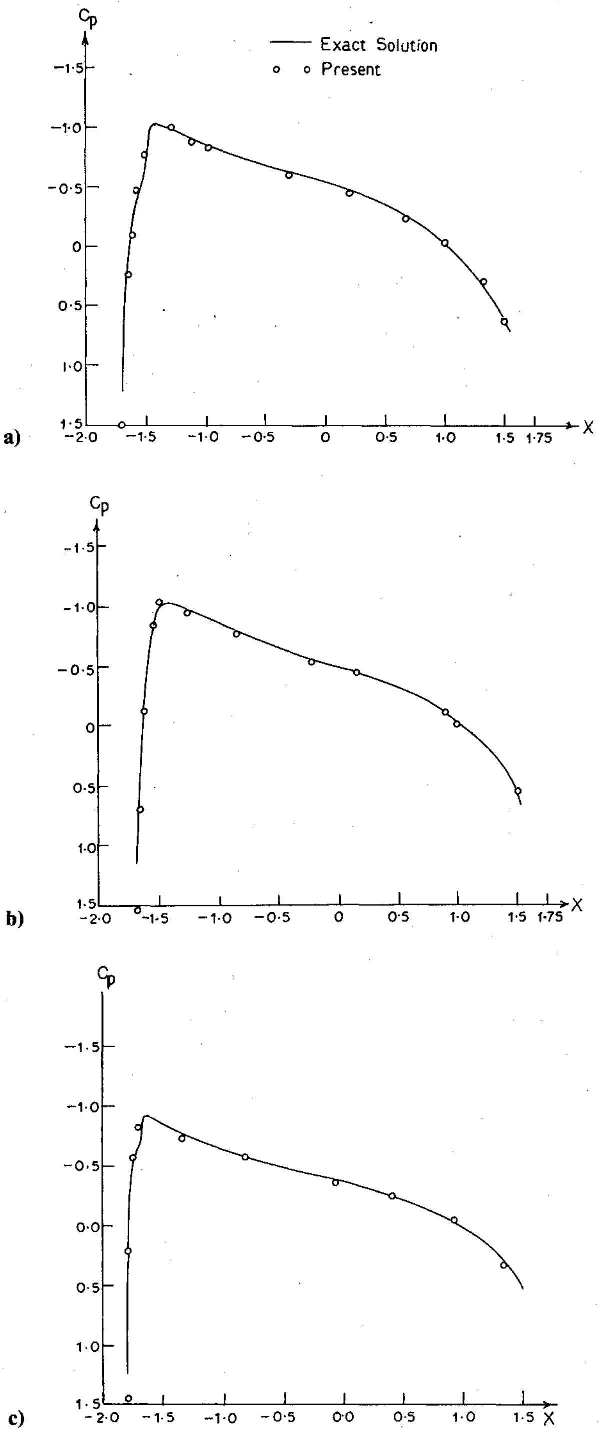

Fig. 1 Surface pressure distribution a) Airfoil section 0.1075-0.6750-1.0500; $M_\infty = 0.7760$. b) Airfoil section 0.1025-0.6750-1.3000; $M_\infty = 0.7557$. c) Airfoil section 0.1150-0.7500-1.2000; $M_\infty = 0.8061$.

$$+\{X_{i+1}-X_i\}\cdot\{2\beta_i-6\alpha_i(X_i-x)\}+\ln\left\{\frac{(X_{i+1}-X)^2+Y^2}{(X_i-X)^2+Y^2}\right\}$$

$$\cdot\left(\frac{3\alpha_i}{2}(X_i-X)^2-\beta_i(X_i-X)+\frac{\gamma_i}{2}-\frac{3}{2}\alpha_i Y^2\right\}$$

$$+\left\{\tan^{-1}\left(\frac{X_{i+1}-X}{Y}\right)-\tan^{-1}\left(\frac{X_i-X}{Y}\right)\right\}$$

$$\cdot\{6\alpha_i Y(X_i-X)-2\beta_i Y\}\Big] \qquad (9)$$

III. Second-Order Edge Correction

A correction is necessary for profiles with a blunt leading edge. We use the second-order edge correction put forward by Nixon and Hancock,[7] which in our notation takes the form:

$$[U_p(X,0)]_{\mathrm{COR}}=\frac{1/k-U_p(X,0)}{[1+(Tk\mathrm{d}f/\mathrm{d}X)^2]^{1/2}}-\frac{1}{k} \qquad (10)$$

where $[U_p(X,0)]_{\mathrm{COR}}$ denotes the corrected linearized solution and $k=(1/M^*_\infty)-1$.

IV. Numerical Results

Since the solution at the body axis is of particular interest, we obtain by setting $Y=0$ in Eq. (4), an approximate shock-free solution

$$U(X,0)=(\sqrt{3}+1)[1-\{1-(\sqrt{3}-1)U_P(X,0)\}^{1/2}] \qquad (11)$$

Three symmetrical quasielliptical Nieuwland profiles[8]: 0.1075-0.6750-1.0500, 0.1025-0.6750-1.3000, and 0.1150-0.7500-1.2000 are selected. $U_p(X,Y)$ at $Y=0$ is then computed using Eq. (9). The second-order correction (10) is then introduced to obtain corrected linearized solution. $U(X,0)$ is then evaluated from Eq. (11). The surface pressure coefficient is computed by the relation

$$C_p=-2U(X,0)[(1/M^*_\infty)-1] \qquad (12)$$

Results are shown in Figs. 1a-c. Comparison with exact solution shows excellent agreement. Computations were carried out with 30 pivotal points on the profile axis, resulting in 30 linear algebraic equations in an equal number of unknowns. Typical CPU time on a Burroughs B 6700 electronic digital computer for computing a profile shape is only 4 s.

In curve fitting, using spline interpolation technique, it is necessary to determine the slope at the end of the curve more or less accurately. Moreover, for the practical utility of the problem it is desirable to employ uniform distribution of mesh points.

Acknowledgment

The author expresses his sincere thanks to Prof. P. Niyogi, of the Indian Institute of Technology, Kharagpur, for suggesting the problem, and for his guidance.

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